

# Surface Delta Interaction in the $g_{7/2} - d_{5/2}$ Model Space

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## Abstract

Using an attractive surface delta interaction we obtain wave functions for 2 neutrons (or neutron holes) in the  $g_{7/2} - d_{5/2}$  model space. If we take the single particle energies to be degenerate we find that the  $g$  factors for  $I = 2, 4$  and  $6$  are all the same  $G(J) = g_l$ , the orbital  $g$  factor of the nucleon. For a free neutron  $g_l = 0$  so in this case all 2 particles or 2 holes'  $g$  factors are equal to zero. Only the orbital part of the  $g$ -factors contribute - the spin part cancels out. We then consider the effects of introducing a single energy splitting between the 2 orbits. We make a linear approximation for all other  $n$  values.

## Surface Delta Interaction

The surface delta interaction (SDI) of Green and Moszkowski [1] and Arvieu and Moszkowski [2] has proven to be a very useful schematic interaction. It can be used to find the hidden simplicity in complex calculations. This interaction has been extensively discussed in Talmi's book [3].

The matrix element of the SDI interaction can be written as follows:

$$\langle [j_1 j_2] SDI [j_3 j_4] \rangle = C_0 f(j_1, j_2) f(j_3, j_4) \quad (1)$$

where we have

$$f(j_1, j_2) = (-1)^{j_2 + \frac{1}{2}} \sqrt{\frac{(2j_1 + 1)(2j_2 + 1)}{(2J + 1)(1 + \delta_{j_1 j_2})}} \times \left\langle j_1 j_2 \frac{1}{2} \left( \frac{-1}{2} \right) \middle| J 0 \right\rangle \quad (2)$$

Note that the expression is separable and so, as indicated by Talmi [3] it is easy to obtain the lowest state wave functions (see Eq. 12.49).

$$\psi^J = N \sum f(j_1, j_2) [j_1 j_2]^J \quad (3)$$

## Wave Functions and $g$ -Factors

We had previously considered a 2-proton hole model for  $^{86}\text{Kr}$  with the relevant orbits being  $f_{5/2}$  and  $p_{3/2}$  [4], stimulated by experiments of Kubartski et al. [5]. We found that with the above SDI interaction and degenerate single particle energies that the  $g$  factors of the  $2^+$  and  $4^+$  states were both equal to  $g_l$ , the orbital  $g$  factor of a proton. The free value of  $g_l$  is one for the proton and zero for the neutron. The spin part does not contribute. We will here consider neutrons in the  $g_{7/2}$  and  $d_{5/2}$  shells and study more intensely the effects of single particle splitting. (Note that a stray factor of 2 under the square root sign of  $f(j_1, j_2)$  which appeared in ref [4] has been removed here. We are now in accord with the expression of Talmi [3]. The numerical results for  $^{86}\text{Kr}$  in ref [4] are consistent with the definition of  $f(j_1, j_2)$  in Eq[2] of this work).

This behaviour (vanishing  $g_s$  contributions) is explained by the pseudo-spin symmetry ideas, developed by Hecht et al. [6] and Arima et al. [7]. Insights into the origin of pseudo-spin symmetry via the Dirac equation has been giving by Ginocchio and Leviatan [8]. Also relevant is the quasi-spin formulation of Kerman [9] and

Talmi's generalized seniority scheme [3]. A. Bohr et al. showed how the spin operator transforms under a pseudo-spin transformation[10]. From the pseudo-spin formulation one can map the orbits  $g_{7/2}$  and  $d_{5/2}$  into  $f_{7/2}$  and  $f_{5/2}$ , thus having all the neutrons in one shell. We further note that the surface delta interaction is a quasi-spin conserving interaction. Note that the expression in Eq(2) depends on  $j$  but not on  $l$ .

We consider 2 neutron particles or 2 neutron holes in either the  $g_{7/2}$  or  $d_{5/2}$  orbital. We choose  $C_0$  to be to be  $-0.2MeV$  We study how the  $G(J)$  for 2 holes or 2 particles depends on the single particle splitting between the  $g_{7/2}$  and  $d_{5/2}$  orbits. We find that the wave function of the 2 particle (2 holes ) state are :

$$BJ77[g_{7/2}g_{7/2}]^J + BJ75[g_{7/2}d_{5/2}]^J + BJ55[d_{5/2}d_{5/2}]^J \quad (4)$$

We show the  $g$  factors  $G(J)$  for 2 particle and also of 2 holes in Tables 1 and 2. the first case we use free single particle  $g$  factors  $g_l = 0$ ,  $g_s = -3.826$  whilst in Table 2 , which we call quenched, we have  $g_l = -0.1$ ,  $g_s = 0.7$   $g_s(free) = -2.678$ .

Table 1:  $G(J)$  as a function of  $E$  with bare input ( $g_l = 0$ ,  $g_s = -3.826$ )

$E(MeV)$	$G(2^+)$	$G(4^+)$	$G(6^+)$
-0.4	0.386	0.390	0.323
-0.3	0.362	0.366	0.247
-0.2	0.314	0.315	0.142
-0.1	0.210	0.205	0.052
0	0	0	0
+0.1	-0.282	-0.294	-0.027
+0.2	-0.496	-0.542	-0.042
+0.3	-0.611	-0.659	-0.051
+0.4	-0.669	-0.708	-0.056

Here the single particle splitting is  $E = \epsilon_{g_{7/2}} - \epsilon_{d_{5/2}}$ . We verify that for  $E = 0$  we get  $G(J) = g_l = 0$  for the case of free neutron values. The spin part does not contribute. Notice how rapidly the  $G(J)$ 's change as a function of  $E$ . This super-sensitivity makes it difficult to pin down the optimum values of the  $G(J)$ 's. An added complication is that we should perhaps use renormalized values of  $g_l$  and  $g_s$  in our analysis. This is shown in Table 2.

Table 2:  $G(J)$  as a function of  $E$  with quenched input ( $g_l = -0.1$ ,  $g_s = -2.678$ )

$E(MeV)$	$G(2^+)$	$G(4^+)$	$G(6^+)$
-0.4	0.160	0.162	0.118
-0.3	0.144	0.146	0.066
-0.2	0.112	0.112	0.005
-0.1	0.041	0.038	-0.065
0	-0.1	-0.1	-0.1
+0.1	-0.290	-0.298	-0.118
+0.2	-0.434	-0.465	-0.128
+0.3	-0.512	-0.544	-0.134
+0.4	-0.551	-0.577	-0.138

Note that now for  $E = 0$  we get  $G(J) = g_l = -0.1$ . Again the renormalized spin part does not contribute.

If we take  $E$  to be minus-infinity we get as asymptotic values for 2 holes :  $G(2^+) = G(4^+) = G(6^+) = g_{7/2}$ . This equals 0.4251 in the free case and 0.2533 in the quenched case.If we take  $E$  to be plus infinity we also get all three  $G$  factors to be the same for 2 particles:  $G(J) = g_{d_{5/2}}$ . For a neutron this equals  $-0.7652$  in the free case and  $-0.6156$  in the quenched case.

In the nucleus  $^{113}\text{Sn}$  the  $J = 7/2^+ - J = 5/2^+$  splitting is  $0.33MeV$ . If we identify this as a single hole nucleus and indeed take the value of  $E$  to be  $-0.33$  then we obtain in the free case  $G(2^+) = 0.371$  ,

$G(4^+) = 0.375$  and  $G(6^+) = 0.274$ . If we use the renormalized values we get  $G(2^+) = 0.150$ ,  $G(4^+) = 0.153$  and  $G(6^+) = 0.0845$ . Notice that in all the cases that we have considered, with  $E$  both positive and negative, we find that  $G(2^+)$  and  $G(4^+)$  are nearly equal.

## Linear Approximation

So far we have considered 2 particles and 2 holes(12 particles) in the model space  $g_{7/2}d_{5/2}$  of neutrons. We now make the speculation that we can obtain the  $G(J)$  values for other  $n$  by a linear approximation. That is to say we assume  $G = G(n = 2) + (G(n = 12) - G(n = 2))/10*(n - 2)$ . This seems reasonable since the more  $g_{7/2}$  neutrons we add the more positive the  $G(J)$ 's should become.

Note that if we go higher in neutron number, we get 2 new orbits for which we can use pseudo  $LS$  coupling  $s_{1/2}$  and  $d_{3/2}$ . If we play the same game, we can now include  $A = 114$  and  $116$ . With single particle energies degenerate, we get zero  $g$  factors. In ref [12] the values for this  $g$  factor are  $+0.138(63)$ ,  $0.0000(64)$ . For completeness, we note that their values for 118, 120, 122, 124 are  $0.000(77)$ ,  $-0.0999(30)$ ,  $0.000(48)$ , and  $-0.097(11)$ .

Table 3,  $G(J)$  as a function of number of valence particles in the linear approximation for  $E = 0.3$  free values.

Table 3: Linear Approximation:  $G(J)$  versus  $n$  with  $E = +0.3$  for 2 particles and  $-0.3$  for 2 holes(bare input)

$n$	$G(2^+)$	$G(4^+)$	$G(6^+)$
2	-0.611	-0.659	-0.051
4	-0.416	-0.454	0.009
6	-0.222	-0.249	0.068
8	-0.027	-0.044	0.128
10	0.167	0.161	0.187
12	0.362	0.366	0.247

Table 4: Linear Approximation:  $G(J)$  versus  $n$  with  $E = +0.3$  for 2 particles and  $-0.3$  for 2 holes(quenched input)

$n$	$G(2^+)$	$G(4^+)$	$G(6^+)$
2	-0.512	-0.544	-0.134
4	-0.381	-0.406	-0.094
6	-0.250	-0.268	-0.054
8	-0.118	-0.130	-0.014
10	0.013	0.008	0.026
12	0.144	0.146	0.066

## Column Vector

As a supplement for Table 1 and Table 2, we list the eigenvalue and the eigenvector for each corresponding  $E$  and  $G$ . Notice that for each  $E$  and  $G$ , there are 2 or 3 (the number depends on the dimensions of SDI-matrix) distinct eigenvalues and corresponding eigenvectors. We will pick the the eigenvector with smallest eigenvalue, for those are the correct ones. We use the same SDI-matrix but different  $g_l$  and  $g_s$  for Table 1 and 2. Therefore, two cases have the same eigensystem.

Table 5: Eigenvalues and Eigenfunctions (column vectors) used to calculate  $G(J)$

Eigensystem for $G(I = 2)$									
$E$	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4
<i>Eigenvalue</i>	-1.039	-0.852	-0.672	-0.508	-0.373	-0.282	-0.231	-0.203	-0.188
$g_{7/2}g_{7/2}$	0.966	0.948	0.914	0.845	0.714	0.536	0.381	0.280	0.215
$g_{7/2}d_{5/2}$	-0.177	-0.212	-0.258	-0.312	-0.350	-0.331	-0.274	-0.218	-0.177
$d_{5/2}d_{5/2}$	0.189	0.238	0.314	0.435	0.606	0.777	0.883	0.935	0.960

Eigensystem for $G(I = 4)$									
$E$	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4
<i>Eigenvalue</i>	-0.926	-0.737	-0.555	-0.389	-0.255	-0.164	-0.116	-0.095	-0.084
$g_{7/2}g_{7/2}$	0.964	0.941	0.891	0.786	0.606	0.404	0.250	0.164	0.118
$g_{7/2}d_{5/2}$	-0.244	-0.310	-0.410	-0.542	-0.639	-0.430	-0.350	-0.304	-0.227
$d_{5/2}d_{5/2}$	0.103	0.136	0.194	0.299	0.474	0.701	0.867	0.938	0.967

  

Eigensystem for $G(I = 6)$									
$E$	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4
<i>Eigenvalue</i>	-0.905	-0.732	-0.579	-0.445	-0.326	-0.215	-0.108	-0.003	0.100
$g_{7/2}g_{7/2}$	-0.892	-0.800	-0.654	-0.498	-0.378	-0.296	-0.240	-0.201	-0.172
$g_{7/2}d_{5/2}$	0.453	0.600	0.756	0.867	0.926	0.955	0.971	0.980	0.985

## Additional Comments

There are 2 known  $g$  factors of relevance for odd-even nuclei. In  $^{113}\text{Sn}$  the  $g$  factor for the  $I = 7/2^+$  state is 0.1737 while in  $^{109}\text{Sn}$  the  $g$  factor of the  $I = 5/2^-$  state is  $-0.4316$ . If we roughly identify these as effective  $g$  factors for  $g_{7/2}$  and  $d_{5/2}$  respectively then for  $E = 0$  above we get an effective neutron  $g_l$  equal to  $-0.0425$ .

The value  $G(6^+)$  in  $^{110}\text{Sn}$  as given by D.A. Volkov et al. is 0.012.[11] Most recently I.M. Allmond et al.[12] measured many  $g$  factors in the tin isotopes. The value of most interest in this work is for  $G(2^+)$  in  $^{112}\text{Sn}$ , namely 0.150(43). In our work this would correspond to  $n = 12$  (or 2 holes). The Allmond et al.value [12] is close to what one gets for  $I = 7/2^+$  in  $^{113}\text{Sn}$  (in the single  $j$  shell all  $g$  factors are the same). It is also close to what we get in our simple model here with renormalized values of  $g_l$  and  $g_s$ . This is for the choice  $C_0 = -0.2$  and  $E = +0.3\text{Mev}$ .

Note that if we go higher in neutron number, we get 2 new orbits for which we can use pseudo  $LS$  coupling  $s_{1/2}$  and  $d_{3/2}$ . If we play the same game, we can now include  $A = 116$ . With single particle energies degenerate, we get zero  $g$  factors. In ref [12] the values for these  $g$  factors is 0.0000(64). For completeness we note that their values for  $A = 114, 118, 120, 122$ , and  $124$  are  $+0.138(63)$ ,  $+0.000(77)$ ,  $-0.0999(30)$ ,  $0.000(48)$ , and  $-0.097(11)$ .

But just fitting one  $g$  factor is not enough. A much more stringent test would be to see if we can get  $G(4^+)$  and  $G(6^+)$  as well. For example, in our surface delta model, we seem to get  $G(4^+)$  almost the same as  $G(2^+)$ . This is either correct or incorrect, and only experiment will tell us. Also in our work, we assume that the  $g$  factors vary linearly with  $n$ . Hence we have e.g. that  $G(J)$  in  $^{110}\text{Sn}$  is smaller than  $G(J)$  in  $^{112}\text{Sn}$ . Only more experimentation will tell us if this is correct or wrong.

In a comprehensive paper by H. Jiang et al. [13] one sees the opposite trend as one goes to lighter tin isotopes. namely an increase in the  $g$  factors as one approaches  $^{102}\text{Sn}$ . They they use an effective value of  $g_l$  that is positive  $g_l = +0.09$ . This is strange, because all theories give a negative value more or less equal and opposite to what they use. The effect of a positive  $g_l$  is to make all the  $g$  factors more positive.

It should also be pointed out that  $E(7/2^+) - E(5/2^+)$  varies very strongly with mass number. The values for  $A = 113, 111, 109, 107, 105, 103, 101$  are respectively  $-332.45, -154.48, +13.48, +151.2, +199.73, +168.0$  and  $-172.2$  or  $+172.3$  keV. The sign for  $A=101$  is in dispute. This makes it difficult to know exactly which  $E$  to choose. Note also the the relevant parameter is really  $E/C_0$ .

We feel that the main virtue of this simple model is that it provides a basis for comparison with more sophisticated calculations and that it provides insight into why many of the  $g$  factors in this region are so small. In this model when degenerate single particles are used all  $g$  factors of the even even nuclei for all spins vanish in the range from  $^{102}\text{Sn}$  and  $^{112}\text{Sn}$ . Going away from zero the  $g$  factors in this model are super-sensitive to the parameters that are used. This will also be true, although less transparent, in more realistic calculations. So we see that in this region of the periodic table both theorists and experimentalists are fighting zero.

## A Brief Look at $^{126}\text{Sn}$

As a counterpoint to the case where we have pseudo  $LS$  partners we here consider a situation where this is not the case. We consider the nucleus  $^{126}\text{Sn}$  where measurements were made by Kumbartzki et al.[14] and where one of us (LZ) was involved. Their result for  $^{126}\text{Sn}$  is  $G(2+) = -0.25(21)$ . They also report the result for  $^{124}\text{Sn}$  as  $-0.36(17)$ . The error bars are quite large. The relevant orbitals are  $h_{11/2}$  and  $d_{3/2}$ . Empirically the 2 orbits are very nearly degenerate e.g. in  $^{125}\text{Sn}$ ,  $I = 3/2^+$  is  $27.5\text{keV}$  above  $I = 11/2^-$  and in  $^{127}\text{Sn}$  the value is only  $5.07\text{keV}$ . We will therefore only show the result for  $E = 0$ .

We find  $G(2) = -0.1096$ ,  $G(4) = -0.5812$ , and  $G(6) = -0.2758$ . Note that we get non-zero values for  $E = 0$  in contrast to what happens when one has pseudo  $LS$  pairs. Note also that  $G(4)$  and  $G(6)$  have not been measured and we predict they should be substantially larger than  $G(2)$ .

The relevant wave functions are:

$$\begin{aligned} I = 0^+ & \quad 0.9737h_{11/2}h_{11/2} + 0.2278d_{3/2}d_{3/2} \\ I = 2^+ & \quad 0.9865h_{11/2}h_{11/2} + 0.4627d_{3/2}d_{3/2} \\ I = 4^+ & \quad 0.5487h_{11/2}h_{11/2} + 0.8360h_{11/2}d_{3/2} \\ I = 6^+ & \quad 0.6757h_{11/2}h_{11/2} + 0.7371h_{11/2}d_{3/2} \end{aligned}$$

We see why  $G(4)$  is much larger than  $G(2)$ . For  $I = 4$  one cannot have a  $d_{3/2} d_{3/2}$  component with its accompanying positive  $g$  factor.

Very recently, the  $E(J = 7/2^+) - E(J = 5/2^+)$  splitting in  $^{107}\text{Sn}$  has been measured by  $g$ . Cerizza et al [15]. They get a value of  $1551\text{keV}$ . This is in line with our choice of  $E$ .

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## Appendix

We here give the  $g$  factor used in our calculations. In Table 4 we use the free values and in Table 5 the renormalized values, both of which are given above. We use the formula

$$G(J) = (g_{7/2} + d_{5/2})/2 + (g_{7/2} - d_{5/2})/(2I(I+1))*(7/2*9/2 - 5/2*7/2).$$

Note that when  $j_1 = j_2$ ,  $G(J)$  is independent of  $J$ . (see Table 6 and 7)

Table 6: Values of  $G(J)$  for the basis states (bare input)

Config.	$G(J)$
$g_{7/2}g_{7/2}$	0.42511
$d_{5/2}d_{5/2}$	-0.76520
$g_{7/2}d_{5/2} \ I = 2$	0.52430
$g_{7/2}d_{5/2} \ I = 4$	0.03825
$g_{7/2}d_{5/2} \ I = 6$	-0.07085

Table 7: Values of  $G(J)$  for the basis states (quenched input)

Config.	$G(J)$
$g_{7/2}g_{7/2}$	0.16647
$d_{5/2}d_{5/2}$	-0.61564
$g_{7/2}d_{5/2} \ I = 2$	0.25331
$g_{7/2}d_{5/2} \ I = 2$	-0.07422
$g_{7/2}d_{5/2} \ I = 6$	-0.14774

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